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Semi-discretization method for solving boundary value problems for parabolic systems

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Annotation: This article discusses the propagation of heat in a three-layer plate. To solve problems, a combination of differential sweeps is used.

Key words: Heat Propagation, Three-Layer Plate, Differential Equations, Differential Sweep.

Consider the propagation of heat in a three-layer plate. The lower and upper plates are heated, and we will consider the distribution of heat in the lower and upper plates vertically, and the middle one horizontally.

The peculiarity of these problems is that the desired functions are included in the equation of the problem in such a way that each of the equations has a "main" unknown function, while the rest are either contained or represented by their own boundary conditions.

Such a problem is called a quasi-two-dimensional problem and can be described by a system of parabolic equations

$$\begin{cases} \frac{1}{\partial z}\left(K_1(z)\frac{\partial u_1(x,z,t)}{\partial z}\right) = A_1(z,t)\frac{\partial u_1(x,z,t)}{\partial t} \\ \frac{1}{\partial x}\left(K(x)\frac{\partial u(x,t)}{\partial x}\right) + \mathrm{H}_1(x)K_1(z)\left.\frac{\partial u_1}{\partial z}\right|_{z=h_1} + \mathrm{H}_2(x)K_2(z)\left.\frac{\partial u_2}{\partial z}\right|_{z=h_0} = A(x,t)\frac{\partial u}{\partial t} \\ \frac{1}{\partial z}\left(K_2(z)\frac{\partial u_2(x,z,t)}{\partial z}\right) = A_2(z,t)\frac{\partial u_2(x,z,t)}{\partial t} \end{cases}$$

(one)

in the area of $\Omega = \{0 < x < 1,\ 0 < z < h_1,\ h_2 < z < h_3,\ 0 < t \le T\}$ (2)

with initial $u(x,o) = u_1(x,z,o) = u_2(x,z,o) = const$

and boundary conditions.

$$\begin{cases} \left.\left[K_1(x)\frac{\partial u}{\partial x} + \beta_1 u(x,t)\right]\right|_{x=0} = \gamma_0 \\ \left.\left[K(x)\frac{\partial u}{\partial x} + \beta_2 u(x,t)\right]\right|_{x=1} = \gamma_1 \\ \left.\left[K_1(z)\frac{\partial u_1}{\partial z} + \beta_{11} u_1\right]\right|_{z=0} = \gamma_{01} \\ \left.\left[K_2(z)\frac{\partial u_2}{\partial z} + \beta_{12} u_2\right]\right|_{z=h_3} = \gamma_{11} \\ u(x,t) = u_1(x,h_1,t) \\ u(x,t) = u_1(x,h_2,t) \end{cases}$$

(3)

where $A(x,t),\ A_1(z,t),\ A_2(z,t),\ H_1(x),\ H_2(x),\ K_1(z),\ K_2(z),\ K(x)$ - given and continuous functions in the range of their arguments. There is a unique solution to the problem [1], [2].

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For an approximate solution, we will apply the following semi-sampling scheme.
The system of equations (1) will be approximated on the straight lines

$$t_j = j\tau,\ \ j=1,...,N,\ \ N=\left[\frac{T}{\tau}\right]$$

The value of the unknown function on the lines $t=t_j$ denote $u_j(x)=u(x,t_j),\ \ u_{1j}(x,z)=u_1(x,z,t_j),\ u_{2j}(x,z)=u_2(x,z,t_j)$

As a result of approximation, we arrive at a system of ordinary differential equations

$$\frac{\partial}{\partial z}\left(K_1(z)\frac{\partial u_{1i}}{\partial z}\right)=A_1(z,t_j)\delta_t u_{ij}$$

$$\frac{d}{dx}\left(K(x)\frac{du_j}{dx}\right)+\mathrm{H}_1(x)K_1(z)\frac{\partial u_{ij}}{\partial z}\bigg|_{z=h_1}+\mathrm{H}_2(x)K_2(z)\frac{\partial u_{2j}}{\partial z}\bigg|_{z=h_2}=A(x,t_j)\delta_t u_{ij}$$

$$\frac{\partial}{\partial z}\left(K_2(z)\frac{\partial u_2}{\partial z}\right)=A_2(z,t_j)\delta_t u_{2j}$$

(four)

Initial conditions (2) and boundary conditions (3) will be replaced, respectively, by the conditions : $u_j(x,o)=u_{1j}(x,z,o)=u_2(x,z,o)=const$

$$\left[K(x)\frac{du_j}{dx}+\beta_1 u_j(x)\right]\bigg|_{x=0}=\gamma_0$$

$$\left[K(x)\frac{du_j}{dx}+\beta_2 u_j(x)\right]\bigg|_{x=1}=\gamma_1$$

$$\left[K_1(z)\frac{\partial u_{1j}}{\partial z}+\beta_{11}u_{1j}(x,z)\right]\bigg|_{z=0}=\gamma_{01}$$

$$\left[K_2(z)\frac{\partial u_{2j}}{\partial z}+\beta_{21}u_{2j}(x,z)\right]\bigg|_{z=h_3}=\gamma_{1i} \quad (5)$$

$$u_j(x)=u_{1j}(x,h_1)$$

$$u_j(x)=u_{2j}(x,h_2)$$

where $\delta_t V_j=\dfrac{V_j-V_{j-1}}{\tau}$ τ - time step.

At every $J=1,2,...,N$ there is a unique solution to problem (4), (5) [1],[2].
The solution of problems (4), (5) will be sought by the differential sweep method.

$$K(x)\frac{du_j}{dx}=C_j(x)u_j(x)+d_j(x)$$

$$K_1(z)\frac{\partial u_{1j}}{\partial z}=C_{1j}(z)u_{1j}(x)+d_{1j}(x,z)$$

$$K_2(z)\frac{\partial u_{2j}}{\partial z}=C_{2j}(z)u_{2j}(x)+d_{2j}(x,z) \quad (6)$$

According to the conditions

$$u_j(1)=\frac{\gamma_1-d_j(1)}{\beta_i+C_j(1)}$$

$$u_{1j}(1)=u_j(x)$$

$$u_{1j}(x,h_1)=u_j(x)$$

$$u_{2j}(x,h_2)=u_j(x) \quad (7)$$

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For unknown functions $C_j(x),\ d_j(x),\ C_{1j}(x) u\ C_{2j}(x),\ d_{2j}(x,z)$ we obtain the following differential equations

$$C_j'(x)=\frac{A_j(x)}{\tau}-\frac{C_j^2(x)}{K(x)}-H_1K_1(h_1)C_{1j}(h_1)-H_2(x)K_2(h_1)C_{2j}(h_2)$$ (eight)

$$d_j^0(x)=\frac{A_j(x)}{\tau}u_{j-1}-H_1(x)d_j(x,h_1)-H_2d(xh_1)-\frac{C_j(x)d_j(x)}{K(x)}$$

With initial condition $C_j^1(0)=-\beta_1,\ \ d_j(0)=\gamma_0$

$$C_{1j}'(z)=\frac{A_1(z)}{\tau}-\frac{C_{1j}^2(z)}{K_1(z)}$$

$$d_{1j}^1(x,z)=-\frac{A_1(z)}{\tau}u_{1j-1}(x,z)-\frac{C_{1j}(z)d_{1j}(x,z)}{K_1(z)}$$

$$C_{1j}(0)=-\beta_{11}\quad d_{1j}(0)=\gamma_{01}$$ (9)

and for $C_{2j}\ \ u\ \ d_{2j}$ function, we get a system of equations

$$C_{2j}'=\frac{A_{1j}(z)}{\tau}-\frac{C_{2j}^2(z)}{K_2(z)}$$

$$d_{2j}'=-\frac{A_{2j}(z)}{\tau}u_{2j-1}-\frac{C_{2j}(x)d_{2j}(x,z)}{K_2(z)}$$

$$C_{2j}(h_2)=-\beta_{12}\quad d_{2j}(h_3)=\gamma_{11}$$ (ten)

Numerical implementation is carried out according to the Runge-Kutta method. We present the solution in two stages (8), (9), (10) - i.e. forward run, then reverse run of tasks (6)-(7) . Let's find $u_j(x),\ u_{1j}(x,1),\ u_{2j}(x,2)$ at every j

It is proved that the approximate solution converges to the exact solution with the speed $0(\tau)$

$$\left|u(x,t_j)-u_j(x)\right|=0(\tau)$$

$$\left|u_{1j}(x,z,t_j)-u_{ij}(x,z)\right|=0(\tau)\quad j=\overline{1,N}$$

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